جامعة عبد المالك السعدي
Université Abdelmalek Essaadi

Pole for Doctoral Studies
Center for Doctoral Studies Sciences and Techniques and Medical Sciences

ANNOUNCEMENT OF DOCTORAL THESIS DEFENSE

Centre des Etudes Doctorales Sciences et Techniques et Sciences Médicales

M. GAHA Anouar

Will present their research work with the aim of earning a Doctorate

Doctoral program: Mathematical Sciences, Physics and New Technologies (SMPNT)
Discipline: Mathematics
Specialty: Algebra and Number Theory

On 23/07/2026 at 12H00 at the Lecture Hall "2", Higher School of Technology of Tetouan, UAE
Under the Theme

On the resolution of Diophantine equations

Front of the jury composed of :

First Name & Last Name	Establishment	Designation
Pr. BOUZELMATE Arij	FS of Tetouan, UAE	President
Pr. DRISSI Ahmed	ENSA of Tangier, UAE	Reviewer
Pr. KACHAD Mohammed	FST of Errachidia, UMI	Reviewer
Pr. TADMORI Abdelhamid	FST of Al Hoceima, UAE	Reviewer
Pr. EL MERZGUIOUI Mhamed	FST of Tangier, UAE	Examiner
Pr. ZOUGARI Tarek	ENSA of Tangier, UAE	Examiner
Pr. MEZROUI Soufiane	EST of Tetouan, UAE	Supervisor

Host Research Structure: Engineering Laboratory for Intelligent Technologies and Transformation (ELITT)

Abstract

The main aim of this thesis is to investigates a systematic study of Diophantine equations and inequalities involving Fibonacci, Lucas, Pell, k-generalized Pell sequences and classical integers. The central methods employed throughout are Baker's theory of linear forms in logarithms of algebraic numbers, various properties of these sequences, continued fractions, congruence techniques, and a version of Baker-Davenport reduction method in Diophantine approximation due to Matveev and Dujella-Pethő. As a first step, for nonnegative integers $n>m$ and a, we showed a conjecture of Erduvan and Keskin by proving the equation $F_n - F_m = p^a$ has no solutions when $a \geq 2$ and primes $p>7$.

More precisely, we confirmed this conjecture by completing analysis for $p=7$ or 13. Similarly, for Lucas numbers, we found all solutions of the equations $L_n - L_m = 5 \cdot 2^a$, $L_n - L_m = 2 \cdot 5^a$ and $L_n - L_m = 10^a$. Cross-sequence proximity problems, for positive integers $n \geq m$, k and s, we were led to study the numbers of $2F_n$, $2L_n$, $2P_n$, sums of two Fibonacci, Lucas and Pell numbers which are close to a power of 3. Equivalently, we solved completely the equation $2P_n^{(k)} = 3^m + t$, where $|t| < 3^{m/2}$, $t \in \mathbb{Z}$. In the sequel, we determined all solutions of the inequalities $|P_n - L_m| < L_m^{1/2}$, $|2F_n - P_m| < P_m^{1/2}$ and $|F_n + F_m - P_s| < P_s^{1/2}$. Pillai's problem involving Pell numbers of the form $3^x - P_n \cdot 2^y = 1$ is solved in positive integers (x,y,n).

Using Catalan's conjecture and modular arithmetic, the exponential equations $13^x + 17^y = z^2$, $13^x + 37^y = z^2$, $17^x + 37^y = z^2$ and $17^x + 91^y = z^2$ have no solutions in nonnegative integers (x,y,z). Thereafter, the equation $43^x + 87^y = z^2$ is solved for consecutive exponents. More general, the equation $(p^n)^x + (3^m p + 2)^y = z^2$ is analyzed in $\mathbb{N}$, with a unique solution identified via elliptic curve methods. Furthermore, the equation $xyz = x + y + z = 2^n$ is studied in finite fields $\mathbb{Z}/p^k\mathbb{Z}$ and in the ring of integers O_K of quadratic field $K = \mathbb{Q}(\sqrt{d})$, where p is an odd prime, $k \geq 1$ and d is a square free integer.

Keywords : Exponential Diophantine equations; Fibonacci, Lucas, Pell and k-generalized Pell numbers; Linear forms in logarithms; Baker's theory; Reduction method; Continued fractions; Pillai's problem; Fermat's equation; Elliptic curves; Factor and congruent method.